AVERAGE ABSORPTION COEFFICIENT FOR OPTICALLY THIN MEDIA

S. P. Detkov

Modifications are considered of the mean Planck absorption coefficient for a section of an absorbing medium adjacent to a source section, taking into account temperature inequalities of blackbody radiation and the absorption cross section of the medium, as well as the effect of the length of the section, which is small. Equations are presented for an analogous coefficient determining self-absorption of radiation by the gas.

Spectral mean absorption coefficients are introduced in order to use gray-gas radiative equations in radiative transfer calculations. In particular, the transmission of a medium is described by an exponential law.

However, the need to use this law results in the average absorption coefficients depending on the ray length x. An evaluation of this dependence allows the so-called optically thin gas approximation to be improved and extended.

Another matter which must be considered is the effect of differences between the radiation source temperature T_1 and that of the absorbing medium T_k. An average Planckian absorption coefficient α_c is used in the literature. It gives the absorption of a blackbody flux by a thin layer of gas. A temperature difference between the blackbody source and the medium is allowed for in [1,2]. The quantity α_c, with a correction factor, is called the modified mean Planckian absorption coefficient. In the well-known gray-gas approximation [2] and elsewhere, the self-absorption of the gas is described by the same coefficient. It is also well known that in this case the actual absorption coefficient α_* is more than an order of magnitude larger than α_c.

The problem is to determine α_c, α_*, and to calculate to what extent they are affected, in the form of corrections, by the quantities T_i, T_k, x_k, x_i. We consider the basic combustion products of hydrocarbon fuels, CO_2 and H_2O. Their spectra are represented in a simplified way in the form of a group of non-overlapping bands.

Restricting ourselves to optically thin media, we represent any direction along a ray path in the form of two adjacent isothermal sections, i and k. The source i can be a blackbody point source. The section k is a segment in the gas. The total pressure is assumed constant, and the field of partial pressure of the radiative components is unchanging. As an argument we take the ray path $x = \int p dl$, m·atm, referred to the partial pressure p. Therefore, all the coefficients denoted by α have dimension $(\mathrm{m.atm})^{-1}$. Local thermodynamic equilibrium is assumed.

The more correct heat transfer equations use the direct absorptance of section k for radiation from section i:

$$a_{ik} = \pi I_{ik} / \varepsilon_i \sigma T_i^4, \qquad I_{ik} = \varepsilon_i \sigma T_i^4/\pi - I_{ik}''.$$

Here ε_i is the emittance of section i; σT^4 is the density of blackbody radiation; I_{ik}, W/m^2.ster is the difference in intensity between the ends of section k; I_{ik}'' is the radiative intensity of the section (point) i, transmitted by section k.

Sverdlovsk. Translated from Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, Vol. 11, No. 1, pp. 12-15, January-February, 1970. Original article submitted May 13, 1969.

If a_{ik} is not defined by the direct equation, an example of which is given below, but the absorption coefficients α are employed, then

$$a_{ik} = 1 - \exp(-\alpha_{ik} x_k), \quad a_{ik}^* = 1 - \exp(-\alpha_{ik}^* x_k). \tag{1}$$

The asterisk denotes the value for self-radiation of the medium. It can be shown that for blackbody radiation a considerably more accurate equation is

$$a_{ik} = \frac{\alpha_{ik}}{\alpha_{ik}^*}\left[1 - \exp(-\alpha_{ik}^* x_k)\right].$$

The equations given are applicable only for fairly thin layers. The expansion $\exp(-u) \approx 1 - u$ is applicable within known limits. Then the quantities α_{ik} can be evaluated by the equations

$$\alpha_{ik} \approx a_{ik}/x_k, \quad \alpha_{ik}^* \approx a_{ik}^*/x_k. \tag{2}$$

The plan for the rest of this paper is: 1) the equations for $a_{ik}*$ is derived, based on a very simple spectral model; 2) the approximate equations (2) are used to determine the coefficients α_{ik} and $\alpha_{ik}*$ as a function of the parameters; 3) the coefficients obtained are recommended for use in the very simple equations (1).

For a group of non-overlapping bands the equation for $a_{ik}*$ takes the form

$$a_{ik}^* = \frac{\pi}{\varepsilon_i \sigma T_i^4} \sum_j I_{0j}(T_i) \int\limits_{\Delta\omega_{*j}} \varepsilon_{\omega i}(T_i, x_i)\, \varepsilon_{\omega k}(T_k, x_k)\, \alpha\omega, \tag{3}$$
$$\varepsilon_\omega = 1 - \exp(-\alpha_\omega x).$$

Here ω (cm^{-1}) is the wave number; ε_ω and α_ω are the spectral values of the emittance and the absorption coefficient; $\Delta\omega_{*j}$ is the width of the narrowest band, chosen so that there is no absorption within it.

If i is a blackbody source point, then $\varepsilon_i = \varepsilon_{\omega i} = 1$. The particular equation for a_{ik} is obtained from Eq. (3). The only source of error in Eq. (3) is associated with choice of the discrete value of the Planck function (I_{0j}).

A discrete value of the Planck function can be used to adjust the band so that the coefficient α_ω varies monotonically. Thereafter, a band profile must be chosen. Analysis shows that for fairly thick layers the absorption is a comparatively weak function of the band profile, but the situation can be quite different in the case considered here, $x_i \approx x_k \approx 0$. We use three different profiles: rectangular, triangular, and symmetrical exponential

$$\alpha_\omega = \alpha_0, \quad \alpha_\omega = \alpha_0(1 - \nu/\Omega), \quad \alpha_\omega = \alpha_0 \exp(-2\nu/\Omega),$$
$$\nu = |\omega - \omega_0|.$$

Here and below Ω is either the profile width or a quantity proportional to it; ω_0 is the position of the profile center where the spectral absorption coefficient has the maximum value α_0.

The corresponding integral band intensities have the form

$$S = \alpha_0\Omega, \quad S = \frac{1}{2}\alpha_0\Omega, \quad S = \alpha_0\Omega. \tag{4}$$

Equations (4) allow us to determine the function $\alpha_0(T)$ in terms of two independent functions $S(T)$ and $\Omega(T)$. According to recent experimental data in [3-5] and elsewhere, we can use a power relation $S \sim T^{\varkappa-1}$, where $\varkappa > 0$ is an exponent giving the dependence of intensity on temperature in a calculation for a single particle; the unit in the exponent takes into account the change in the number of particles associated with a constant expansion. In thin layers, bands with fundamental frequencies play an exceptional role. For these $\varkappa \approx 0$. The relation $\Omega \sim T^m$ was assumed for the half-width or the width of all the bands. From spectroscopic data $m \approx 0.5$.

The result is a unique temperature dependence for the values of α_0. Independently of the profile we have

$$\alpha_0 \sim T^u, \qquad u = \varkappa - m - 1 \quad (\varkappa \approx 0)$$
$$\alpha_{0k}(T_k) = \alpha_{0i}(T_i)\,\xi^u, \qquad \xi = T_k / T_i . \tag{5}$$

Here and below we use the relations

$$\alpha_c = \frac{\pi}{\sigma T^4} \sum_j I_{0j} \int_{\Delta\omega_j} \alpha_\omega \, d\omega, \qquad S_j = \int_{\Delta\omega_j} \alpha_\omega \, d\omega$$
$$\alpha_c \alpha_* = \frac{\pi}{\sigma T^4} \sum_j I_{0j} \int_{\Delta\omega_j} \alpha_\omega^2 \, d\omega, \qquad \frac{S_j^2}{\Omega_j} \sim \int_{\Delta\omega_j} \alpha_\omega^2 \, d\omega . \tag{6}$$

Here $\Omega\omega$, (in contrast with Ω_j) is the total band with.

We consider the absorption of a blackbody ray. The quantity a_{ik} was obtained from Eq. (3) under the conditions

$$\varepsilon_i = \varepsilon_{i\omega} = 1, \qquad x_k \approx 0, \qquad \varepsilon_{\omega k} \approx \alpha_{\omega k} x_k - {}^1/_2\, \alpha_{\omega k}^2 x_k^2 .$$

Then

$$a_{ik} \approx \frac{\pi}{\sigma T_i^4} \sum_j I_{0j}(T_i) \int_{\Delta\omega_j} \left[\alpha_{\omega k} x_k - \frac{1}{2} \alpha_{\omega k}^2 x_k^2\right] d\omega .$$

Use of Eqs. (2) and (4)-(6) leads to the result

$$\frac{\alpha_{ik}(T_i, T_k, x_k)}{\alpha_c(T_i)} \approx \xi^{u+m} \left(1 - \frac{1}{2} \alpha_*(T_i)\, x_k \xi^u\right). \tag{7}$$

It is noteworthy that the quantity α_{ik} does not depend on the profile shape nor on the inequality $T_i \gtreqless T_k$.

Next we calculate the self-absorption by the gas. It is not difficult to show that if the band profiles of the hotter section are rectangular, then the band profiles of the other section are not important. With profiles identical for the two sections, the results are different.

In the derivation of α_{ik*} from Eq. (3), we restrict ourselves to two terms of the series in the expansions of $\varepsilon_{\omega i}$ and $\varepsilon_{\omega k}$

$$\varepsilon_{\omega i} \varepsilon_{\omega k} \approx \alpha_{\omega i} \alpha_{\omega k} x_i x_k .$$

Correspondingly we put $\varepsilon_i \approx \alpha_c x_i$. Equation (3), taking into account Eq. (2), becomes simpler:

$$\alpha_c(T_i)\, \alpha_{ik}{}^* \approx \frac{\pi}{\sigma T_i^4} \sum_j I_{0j}(T_i) \int_{\Delta\omega_{*j}} \alpha_{\omega i} \alpha_{\omega k} \, d\omega .$$

The limit of integration for $\Delta\omega_{*j}$ has the peculiarity that, for a limited band (rectangular or triangular), the integration is carried out over the narrowest band after the maxima have been made to coincide. The following results are obtained:

$$\frac{\alpha_{ik}{}^*(T_i, T_k)}{\alpha_*(T_i)} = \xi^{u+m} f_1(\xi) \qquad \text{for} \quad \xi \leqslant 1$$
$$\frac{\alpha_{ik}{}^*(T_i, T_k)}{\alpha_*(T_i)} = \xi^u f_2(\xi) \qquad \text{for} \quad \xi \geqslant 1 .$$

We present expressions for the functions f_1 and f_2 of Eqs. (8) for different band profiles

rectangular $f_1 = 1, f_2 = 1$
triangular $f_1 = {}^1/_2(3 - \xi^m), f_2 = {}^1/_2(3 - \xi^{-m})$
exponential $f_1 = 2(1 + \xi^m)^{-1}, f_2 = 2\xi^m(1 + \xi^m)^{-1}$

For $\xi = 1$ and any profile, the correct result $\alpha_{ik*} = \alpha_*$ is obtained. For $\xi^m \ll 1$ or $\xi^m \gg 1$ the triangular and the exponential profiles give results which are greater by factors of 1.5 and 2 than for the rectangular profile. In the case $\xi^m \gg 1$ this cannot have much importance because of the weak absorption ($u \approx -1.5$, $\xi^u \ll 1$, $\alpha_{ik*} \ll \alpha_*$). In contrast to the rectangular and triangular profiles, the exponential profile has distant wings, similar to an actual profile. But it also has the defect that the dependence of α_{ik*}/α_* on ξ does not change form in passing through the point $\xi = 1$. This change should be in agreement with the representation of the spectral "windows," which approximately reflects the real situation. Possibly the triangular profile is the most realistic of those considered in our work.

It is considerably more complex to evaluate the dependence of $\alpha_{ik*}(x_k)$, since we must take a minimum of three terms in the series for $\varepsilon_{\omega k}$.

The coefficients α_c and α_* are based on the equations for α_{ik} and α_{ik*}. They can be determined from Eq. (6) with spectroscopic data included. Another method is to use the integrated emittances $\varepsilon(x,T)$ published for CO_2 and H_2O in the well-known monographs [6]. The equations

$$\alpha_c = (\partial\varepsilon / \partial x)_{x=0}, \qquad \alpha_c\alpha_* = -(\partial^2 \varepsilon / \partial x^2)_{x=0}$$

are used.

Data on α_c are given in [6-8] and elsewhere. Values of α_* are given in [6] in implicit form. The accuracy of α_c, and particularly, of α_*, is not sufficient for CO_2 and H_2O.

In conclusion we note that the relation for $\alpha_{ik}(T_i, T_k)$, represented in Eq. (7) with $x_k = 0$, was obtained in complete agreement with [1,2,9]. We note also that the exponent m does not come in, since $u + m = \varkappa - 1$. Estimates of the effect of the path length according to Eq. (7), and of the dependence of $\alpha_{ik*}(T_i, T_k)$, represented by Eqs. (8), have been obtained for the first time.

LITERATURE CITED

1. R. D. Cess and P. Mighdoll, "Modified Planck mean coefficients for optically thin gaseous radiation," Internat. J. Heat Mass Transfer, 10, no. 9, p. 1291, 1967.
2. R. D. Cess, P. Mighdoll, and S. N. Tiwari, "Infrared radiative heat transfer in nongray gases," Internat. J. Heat Mass Transfer, 10, no. 11, p. 1521, 1967.
3. J. C. Breeze, C. C. Ferriso, C. B. Ludwig, and W. Malkmus, "Temperature dependence of the total integrated intensity of vibrational-rotational band systems," J. Chem. Phys., 42, no. 1, p. 402, 1965.
4. J. C. Breeze and C. C. Ferriso, "Shock wave integrated intensity measurements of the 2.7 micron CO_2 band between 1200° and 3000° K," J. Chem. Phys., 139, no. 10, p. 2619, 1963.
5. C. C. Ferriso and C. B. Ludwig, "Spectral emissivities and integrated intensities of the 1.87-, 1.38-, and 1.14-micron H_2O bands between 1000° and 2400° K," J. Chem. Phys., 41, no. 6, p. 1668, 1964.
6. A. S. Nevskii, Heat Transfer in Open-Hearth Furnaces [in Russian], Metallurgizdat, Moscow, 1963.
7. S. P. Detkov and V. N. Girs, "Mechanization of radiation calculations in carbon dioxide," Izvestiya VUZ, Chernaya metallurgiya, no. 2, p. 162, 1967.
8. M. M. Abu-Romia and C. L. Tien, "Appropriate mean absorption coefficients for infrared radiation of gases," Trans. ASME, ser. C. J., Heat Transfer, 89, no. 4, 1967.
9. S. S. Penner, Quantitative Molecular Spectroscopy and Gas Emissivities [Russian translation], Izd-vo inostr. lit., Moscow, 1963.